4.3.3 Properties

Chord

- Chords are equidistant from the center of a circle if and only if they are equal in length.
- The perpendicular bisector of a chord passes through the center of a circle; equivalent statements stemming from the uniqueness of the perpendicular bisector are:
    - A perpendicular line from the center of a circle bisects the chord.
    - The line segment through the center bisecting a chord is perpendicular to the chord.
- If a central angle and an inscribed angle of a circle are subtended by the same chord and on the same side of the chord, then the central angle is twice the inscribed angle.
- If two angles are inscribed on the same chord and on the same side of the chord, then they are equal.
- If two angles are inscribed on the same chord and on opposite sides of the chord, then they are supplementary.
    - For a cyclic quadrilateral, the exterior angle is equal to the interior opposite angle.
- An inscribed angle subtended by a diameter is a right angle.
- The diameter is the longest chord of the circle.
    - Among all the circles with a chord AB in common, the circle with minimal radius is the one with diameter AB.
- If the intersection of any two chords divides one chord into lengths a and b and divides the other chord into lengths c and d, then $ab = cd$.
- If the intersection of any two perpendicular chords divides one chord into lengths a and b and divides the other chord into lengths c and d, then $a^2 + b^2 + c^2 + d^2$ equals the square of the diameter.
- The sum of the squared lengths of any two chords intersecting at right angles at a given point is the same as that of any other two perpendicular chords intersecting at the same point, and is given by $8r^2 - 4p^2$ (where r is the circle's radius and p is the distance from the center point to the point of intersection).

- The distance from a point on the circle to a given chord times the diameter of the circle equals the product of the distances from the point to the ends of the chord.

Tangent

- A line drawn perpendicular to a radius through the end point of the radius lying on the circle is a tangent to the circle.
- A line drawn perpendicular to a tangent through the point of contact with a circle passes through the center of the circle.
- Two tangents can always be drawn to a circle from any point outside the circle, and these tangents are equal in length.
- If a tangent at A and a tangent at B intersect at the exterior point P, then denoting the center as O, the angles $\angle BOA$ and $\angle BPA$ are supplementary.
- If AD is tangent to the circle at A and if AQ is a chord of the circle, then $\angle DAQ = \frac{1}{2}$ arc(AQ).

Theorems

- The chord theorem states that if two chords, CD and EB, intersect at A, then $AC \times AD = AB \times AE$.
- If two secants, AE and AD, also cut the circle at B and C respectively, then $AC \times AD = AB \times AE$. (Corollary of the chord theorem.)
- A tangent can be considered a limiting case of a secant whose ends are coincident. If a tangent from an external point A meets the circle at F and a secant from the external point A meets the circle at C and D respectively, then $AF^2 = AC \times AD$. (Tangent-secant theorem.)
- The angle between a chord and the tangent at one of its endpoints is equal to one half the angle subtended at the center of the circle, on the opposite side of the chord (Tangent Chord Angle).
- If the angle subtended by the chord at the center is 90 degrees then $\ell = r\sqrt{2}$, where ℓ is the length of the chord and r is the radius of the circle.
- If two secants are inscribed in the circle as shown below, then the measurement of angle A is equal to one half the difference of the

measurements of the enclosed arcs (DE and BC). That is

$2\angle CAB = \angle DOE - \angle BOC$, where O is the center of the circle. (Secant-secant theorem.)

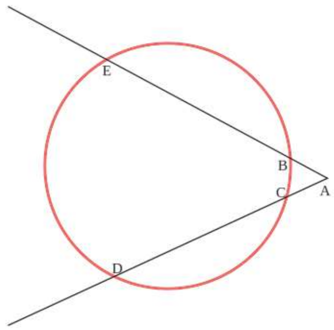

Figure: Secant-secant theorem

Inscribed Angles

An inscribed angle (examples are the blue and green angles in the figure) is exactly half the corresponding central angle (red). Hence, all inscribed angles that subtend the same arc (pink) are equal. Angles inscribed on the arc (brown) are supplementary. In particular, every inscribed angle that subtends a diameter is a right angle (since the central angle is 180 degrees).

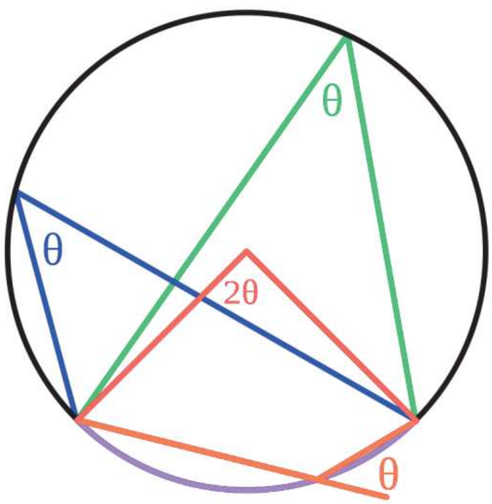

Figure: Inscribed angle theorem